

# Aggregate Implications of Merger Policy

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## Abstract

To what degree, should antitrust agencies allow firms to merge? While this question has been frequently investigated, lifecycle of industries has been often neglected in the context. To take into account the importance of lifecycle consideration in merger analysis, we build a multi-industry macroeconomic model with oligopolistic competition. The model captures a typical lifecycle of industries: an industry starts with a few firms, then have many firms, and experiences a drop in the number of firms, and a smaller number of firms survive. When the industry is old, merger is detrimental to consumers because it allows firms to obtain large market power and limit production. On the other hand, merger opportunities induce entry at the early stage of an industry lifecycle. Death and birth of industries generate heterogeneity of industries in terms of age. Since households consumer goods from industries some of which are young and others of which are old, they care about outputs from both young and old industries. If merger analysis of industries is conducted at later periods of the lifecycle, when antitrust issues are typically recognized, the antitrust authority finds tougher regulation is more favorable than households do, leading bias towards older industries.

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# 1 Introduction

To what degree, should antitrust agencies allow firms to merge? Although this is a classical question in economics, few studies take into account the fact that industries grow overtime and located at different stages: some industries are just born and on the path of persistent growth, and others are matured with saturated growth. Given such consideration, how should we think of merger policy?

To answer the question, we build a multi-industry macroeconomic model where each industry features dynamics and oligopolistic competition. The model features the life cycle of an industry – an industry starts without any capital and firms gradually accumulate capital. In each industry, firms faces oligopolistic competition. Merger is modeled as an exogenous opportunity to acquire capital from other firms, and it affects market share more than the internal accumulation of capital does. Under the presence of firm level overhead labor, merger creates more market power and save the overhead labor.

The model can generate a typical lifecycle of industry: it starts with a few firms, the number of firms builds up, and it experiences a shakeout –a large drop in the number of firms– and a smaller number of firms survive. The non-monotonic pattern in the industry lifecycle is documented in the literature of the industry lifecycle such as Gort and Klepper (1982) and Jovanovic and MacDonald (1994). To our knowledge, our work is the first to study how the lifecycle patterns is related to merger policy.

The first contribution of our work is to show loose merger policy attracts more entry at the early stage of the lifecycle, and therefore explicitly capturing the lifecycle of industries introduces differential effect of merger policy over the lifecycle. While many existing industry analyses incorporate endogenous entry decision, they do not take into account the lifecycle of industries. When the industry is old enough, incumbents firms already accumulated capital and entry is not as encouraged as it is in the early stage of the lifecycle, when entry incentive is more sensitive to change in the market structure. Given that entry is more encouraged in the beginning of the industry, we find that merger policy has different implications over the lifecycle. Especially, If merger policy is evaluated at the later stage of the lifecycle, as a typical industry analysis is conducted, a loss in consumer surplus is highlighted.

The second contribution is to clarify how the planner's view and an industry regulation view disagree with each other. The planner faces many industries which are located at different stages of the lifecycle, therefore she cares about both younger and older industries. On the other hand, a typical industry analysis maximizes expected present value consumer surplus given the state of the industry and prices, and discount using market interest rate, and conducted at later stage. The tendency that merger analysis is conducted sufficiently after the birth of the industry will be virtually putting more weight on older industries from the

planner’s perspective. We find that if the industry analysis were conducted at the very beginning of the lifecycle stage, the industry evaluation would be closer to the planner’s one.

## 2 Related literature

Our study is primarily related to studies of dynamic welfare analysis of merger policy. For example, based on Ericson and Pakes (1995) and Pakes and McGuire (1994)’s oligopolistic competition framework, Gowrisankaran (1995, 1999) pioneered an oligopolistic competition model with endogenous merger. Many works followed him to study merger policy in dynamic setups of Ericson-Pakes-McGuire framework. A recent example is Mermelstein, Nocke, Satterthwaite and Whinston (2019), a numerical theory of duopoly with merger and innovation.

There are two papers in the literature of dynamic merger analysis which are closely related to our work. Igami and Uetake (2019) is the closest to our paper and they study merger policy with a setup with typical lifecycle pattern using data from the hard disk drive industry. However, focus of the merger analysis was periods after the hard disk drive industry experienced a shakeout, and they did not study how the lifecycle pattern of the hard disk industry would be affected under different merger policy regime. The other work is Gowrisankaran and Holmes (2004) which study how merger opportunity affects entry incentive and resulting long-run market structure, but they did not focus on the lifecycle of the industry.

The non-monotonic pattern of the number of firms in industries are confirmed across many industries and summarized in Gort and Klepper (1982), Klepper and Graddy (1990), and Klepper and Simons (1997). These papers observe that new industries commonly experience exodus of firms after a buildup in the number of firms, which is called a “shakeout”. Jovanovic and MacDonald (1994) study the U.S. automobile tire industries and attributed to technological innovation. This type of explanation is also used in Hünermund, Schmidt-Dengler and Takahashi (2014) and Igami (2018). Industry dynamics in this paper are constructed so that they captures these lifecycle patterns, but different from Jovanovic and MacDonald (1994), our model simply use variety expansion of firms to generate the typical lifecycle pattern of industries.

We also make a contribution to a literature on what basis merger policy needs to be evaluated. Traditional merger analysis is conducted based on static tradeoff (e.g. Williamson (1968)). Berry and Pakes (1993) argue that dynamic consideration may overturn a static analysis. Nocke and Whinston (2010) show evaluating mergers using myopic measure of consumer surplus and total surplus can be consistent with present-value welfare criteria. Our work provides how these criteria disagree with the planner’s criterion.

While our model is based on the framework developed by Ericson and Pakes (1995) and Pakes and McGuire

(1994), it deviates from their framework and introduces mechanics of firm growth from Klette and Kortum (2004) and Luttmer (2011)<sup>1</sup>. With such mechanics, our model’s investment technology naturally exhibit “merger-neutral” property developed by Mermelstein, Nocke, Satterthwaite and Whinston (2019). Our paper provide a generic standard environment that generate “merger-neutral” innovation technology that is easy to be extended to a general setup.

Our work is also related to a literature on aggregate implication of merger and acquisition. Jovanovic and Rousseau (2002), Jovanovic and Rousseau (2008) and Xu (2017) regard merger as capital reallocation process. David (2020) thinks of merger as search-and-matching process of creating synergies. While we share the same objective to evaluate the role of merger and acquisition from an aggregate perspective, our main focus on merger and acquisition is change in market power and cost, and its effect on industry lifecycles.

Recent literature on competition and macroeconomic trends described in De Loecker, Eeckhout and Unger (2020), Autor, Dorn, Katz, Patterson and Van Reenen (2020), and Gutiérrez and Philippon (2019). Weiss (2020) and Cavenaile, Çelik and Tian (2020) study macroeconomic trend of the rising markup and concentration using quantitative models of oligopolistic competition. Helpman and Niswonger (2020) also study the trend by building a model where firms keep expanding product variety. These papers abstract from the lifecycle of industries and mergers, which are focus on our study.

Lastly, this paper is related to the literature on allocative efficiency across firms and industries. Dixit and Stiglitz (1977) show that monopolistic competition achieves aggregate efficiency if demand structure is CES and households do not enjoy leisure. Many authors follow them to characterize and quantify the allocative efficiency. Examples include Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Opp, Parlour and Walden (2014), Holmes, Hsu and Lee (2014), Edmond, Midrigan and Xu (2015), Asker, Collard-wexler and De Loecker (2016), Bilbiie, Ghironi and Melitz (2019), and Peters (2020). In contrast to these works, our setup takes into account both industry dynamics lifecycle and oligopolistic competition, and put emphasis on merger policy.

### 3 Model

There is a unit continuum of dynastic households with no population growth. Preference over households consumption  $C_t > 0$  is given by

$$\int_0^\infty e^{-\rho t} \ln C_t dt$$

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<sup>1</sup>This can be considered of a dynamic extension to Nocke and Schutz (2018) with homogeneous technology.

where  $\rho > 0$  is time preference. Consumption is a composite good made up of a continuum of differentiated goods,

$$C_t = \left[ \int_0^1 C_{j,t}^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \quad (1)$$

where  $\eta (> 1)$  is the elasticity of substitutions across industry goods  $\{c_{j,t}\}_{j \in [0,1]}$ . Industry output  $C_{j,t}$  is itself a composite good, but now with a finite number of differentiated goods,

$$C_{j,t} = \left[ \sum_{n=1}^{N_{j,t}} c_{n,j,t}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}},$$

where  $\xi (> \eta)$  is the elasticity of substitution of differentiated goods within a industry,  $N_{j,t}$  is the number of differentiated goods in industry  $j$  time  $t$ .

Households do not enjoy utility from leisure. Security markets are complete. Households own all the asset in this economy and face a standard present-value budget constraint with no-Ponzi condition. Assets in industry  $j$  are an industry-specific fixed factor and  $N_{j,t}$  distinct blueprints to produce differentiated goods in industry  $j$ . Households can create a new blueprints by combing labor and the industry  $j$  fixed factor.  $N_{j,t}$  blueprints in industry  $j$  are controlled by firms and a firm with more than one blueprints can coordinate production decisions of the blueprints it controls. Firms can't control blueprints across industries.

### 3.1 Blueprints

A blueprints is capital to produce a differentiated commodity in a particular industry. Except that each blueprint produces a distinct commodity from each others, production technologies of the blueprints are the same and households equally enjoy all the differentiated goods. Given a blueprint for differentiated commodity  $n$  industry  $j$ , it takes one unit of labor for  $z$  unit of the differentiated commodity produced. That is to say,

$$c_{n,j,t} = z \ell_{n,j,t}.$$

Blueprints are accumulated endogenously in two ways. An industry-specific fixed asset can be combined with  $\ell$  unit of labor to create one blueprint randomly at the Poisson rate  $E(\ell) = e_0 \ell^{e_1}$  ( $e_0 > 0, e_1 \in (0, 1)$ ). Blueprints can be also produced by randomly replicating individual blueprints at the Poission rate  $\mu(\ell)$ , where  $\ell$  is the amount of labor combined with a particular blueprint and  $\mu(\ell) = \mu_0(\ell)^{\mu_1}$  ( $\mu_0 > 0, \mu_1 \in (0, 1)$ ).

Two types of depreciation shocks to blueprints are considered. Individual blueprints die at the random rate

$\delta$ , and these shocks are independent across blueprints. In addition, all the existing blueprints in industry  $j$  become obsolete at the rate  $\lambda$ . The shocks are independent across industries but common within an industry. The obsolescence shock captures the death and birth of an industry. Upon the arrival of an obsolescence shock, households stop enjoying consumption from industry  $j$  and the specific blueprints and fixed asset become obsolete. At the same time, a new industry-specific fixed asset arrives. We index the new industry again  $j$  for simplicity.

### 3.2 Consumer Demands

Households who face commodity price  $p_{n,j,t}$  make consumption choices such that

$$c_{n,j,t} = \left( \frac{p_{n,j,t}}{P_{j,t}} \right)^{-\xi} C_{j,t}, \quad C_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\eta} C_t$$

where

$$P_{j,t} = \left[ \sum_{n=1}^{N_{j,t}} p_{n,j,t}^{1-\xi} \right]^{\frac{1}{1-\xi}}, \quad P_t = \left[ \int_0^1 P_{j,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

Therefore,  $P_{j,t}$  is the price index for the industry- $j$  composite good and  $P_t$  is the overall price index.  $c_{n,j,t}$  can be also written by

$$\frac{p_{n,j,t}}{P_{j,t}} = \left( \frac{c_{n,j,t}}{C_{j,t}} \right)^{-\frac{1}{\xi}} \left( \frac{C_{j,t}}{C_t} \right)^{-\frac{1}{\eta}}$$

Since there are a lot of industries, firms for industry  $j$  take  $P_t$  and  $C_t$  as given when they decide on its production and pricing.

### 3.3 Firm's problem

A firm is a collection of blueprints in an industry and maximizes the present-value by selling goods, replicating its blueprints, and acquiring blueprints from other firms. It also needs to hire a fixed amount of labor as long as it operates. The fixed cost is meant to capture the type of labor cost firms have to pay to operate independent of its size, such as human resource.

We assume that firms take Markov strategy to maximize their present value<sup>2</sup>. A firm's strategy is production decision, replication decision, and merger decision indexed by the number of blueprints it owns and firm distribution, which is to be specified later. A firm's optimization is to choose a strategy to maximize its present value given strategies of other firms given aggregate prices and quantities.

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<sup>2</sup>This assumption excludes, for example, a strategy that results in a cartel among incumbents firms.

### 3.4 Production decision

The production decision is formulated as below. Let  $\mathcal{N}_{j,t} = \{1, 2, \dots, N_{j,t}\}$  a set of distinct blueprints in industry  $j$ . There are  $I_{j,t}$  number of firms in industry  $j$  and firm  $i \in \{1, 2, \dots, I_{j,t}\}$  control  $\mathcal{N}_{i,j,t} \subset \mathcal{N}_{j,t}$  blueprints. Let  $N_{i,j,t} = |\mathcal{N}_{i,j,t}|$  denote the number of distinct blueprints firm  $i$  controls over.  $\{\mathcal{N}_{i,j,t}\}_{i \in I_{j,t}}$  is a partition of  $\mathcal{N}_{j,t}$ . Firm  $i$  maximizes the static joint profits from  $\mathcal{N}_{i,j,t}$  by choosing quantities for each blueprints  $\{c_{n,j,t}\}_{n \in \mathcal{N}_{i,j,t}}$  given quantities chosen by other firms  $\{c_{n,j,t}\}_{n \in \mathcal{N}_{-i,j,t}}$ ,

$$\begin{aligned} & \max_{\{c_{jk}, p_{jk}, \ell_{jk}\}_{k \in \mathcal{K}_{ji}}} \sum_{n \in \mathcal{N}_{i,j,t}} \left( \frac{p_{n,j,t}}{P_t} c_{n,j,t} - \frac{w_t}{z} c_{n,j,t} \right) \\ & \text{s.t. } \frac{p_{n,j,t}}{P_t} = \left( \frac{c_{n,j,t}}{C_{j,t}} \right)^{-\frac{1}{\xi}} \left( \frac{C_{j,t}}{C_t} \right)^{-\frac{1}{\eta}} \quad (\forall n \in \mathcal{N}_{i,j,t}) \\ & C_{j,t} = \left[ \sum_{n=1}^{N_{j,t}} c_{n,j,t}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} \\ & \text{given } \{c_{n,j,t}\}_{n \in \mathcal{N}_{-i,j,t}}, C_t, P_t, w_t. \end{aligned}$$

where  $\mathcal{N}_{-i,j,t} = \mathcal{N}_{j,t} \setminus \mathcal{N}_{i,j,t}$ .

The first order conditions read all the blueprints under the control of the same firm should be produced and priced by the same quantities and prices in an equilibrium. The conditions also characterize resulting prices of the differentiated commodities under the control of firm  $i$ ,

$$p_{n,j,t} = \frac{\varepsilon(s_{i,j,t})}{\varepsilon(s_{i,j,t}) - 1} \frac{w}{z} \quad (\forall n \in \mathcal{N}_{i,j,t})$$

where  $s_{i,j,t}$  is expenditure share of firm  $i$ ,

$$s_{i,j,t} = \frac{\sum_{n \in \mathcal{N}_{i,j,t}} p_{n,j,t} y_{n,j,t}}{\sum_{n \in \mathcal{N}_{j,t}} p_{n,j,t} y_{n,j,t}}$$

and  $\varepsilon(s_{i,j,t})$  is given by

$$\frac{1}{\varepsilon(s_{ji})} = \frac{1}{\eta} s_{ji} + \frac{1}{\xi} (1 - s_{ji}).$$

The interpretation of this formula is that firm  $i$  faces smaller price elasticity when its market share is larger, and it faces larger elasticity when the share is smaller. Here, the market share of firm  $i$  is larger when it controls more distinct blueprints.

Let  $\Gamma_{j,t} \in \mathbb{N}^\infty$  be a distribution of  $N_{i,j,t}$  in industry  $j$ . That is to say,  $\Gamma_{j,t}$  records the number of firms

with one blueprints, the number of firms with two blueprints, and so on. Given the structure above, profits of firm  $i$  controlling  $N_{i,j,t}$  is now written by  $\pi(N_{i,j,t}; \Gamma_{j,t})$ .

### 3.5 New blueprints creation by replication

A firm in industry  $j$  creates another industry- $j$  specific blueprint by randomly replicating the blueprints it controls over. Specifically, the firm replicates a distinct blueprints  $n \in \mathcal{N}_{i,j,t}$  it controls over at the Poisson rates  $\mu(\ell_{n,j,t}) = \mu_0(\ell_{n,j,t})^{\mu_1}$ . If replication succeeds, the firm holds another blueprint and the industry state  $\Gamma_{j,t}$  becomes  $\Gamma_{j,t} - \boldsymbol{\nu}_{N_{i,j,t}} + \boldsymbol{\nu}_{N_{i,j,t}+1}$ , where  $\boldsymbol{\nu}_N$  is a unit vector with its  $N$ -th element one. Since all the blueprints  $\mathcal{N}_{i,j,t}$  are symmetric, the replication rate of the firm can be written by

$$M(\ell_{i,j,t}^{rd}, N_{i,j,t}) = \sum_{n \in \mathcal{N}_{i,j,t}} \mu(\ell_{n,j,t})$$

where  $\ell_{i,j,t}^{rd} = \sum_{n \in \mathcal{N}_{i,j,t}} \ell_{n,j,t}$  is total labor input for replication for firm  $i$ .

### 3.6 Blueprints acquisition by a merger opportunity

A firm periodically receives a merger opportunity with a Poisson rate  $\theta$ , and it may purchase another firm in the same industry or stay away from the deal.

Procedure of merger decision is specified in the following way. When a merger opportunity arrives, a firm can make an offer only if there are firms other than  $i$  in industry  $j$ . If not, nothing happens to firm  $i$ . If there are firms other than  $i$ , the firm privately observes valuation shocks  $\{\epsilon_m\}$  for each potential target  $m \in T_{i,j,t}$ , where  $T_{i,j,t} \subset \Gamma_{j,t} \setminus \{i\}$  is a set of the other firms. The acquirer may choose not to make any offer, and it receives nothing. If the acquirer decides to purchase  $m$ , it makes a take-or-leave-it offer to  $m$ , and target  $m$  always accepts the offer. The acquirer pays a present value of the firm to target  $m$ , receives a valuation shock  $\epsilon_m$ , and obtain all the blueprints target  $m$  owns. Target  $m$  exits from industry  $j$ .

The maximization problem of an acquirer after observing  $\{\epsilon_m\}_{m \in \{0\} \cup T}$  is given by

$$\max \left\{ 0, \max_{m \in T_{i,j,t}} \{V(N_{i,j,t} + N_{m,j,t}; \Gamma'_{j,t}) - V(N_{i,j,t}; \Gamma_{j,t}) - V(N_{m,j,t}; \Gamma_{j,t}) + \epsilon_m\} \right\}$$

where  $\Gamma'_{j,t} = \Gamma_{j,t} + \boldsymbol{\nu}_{N_{i,j,t} + N_{m,j,t}} - \boldsymbol{\nu}_{N_{i,j,t}} - \boldsymbol{\nu}_{N_{m,j,t}}$

where  $\boldsymbol{\nu}_N$  is a unit vector with  $N$ -th element one,  $V(N_{i,j,t} + N_{m,j,t}; \Gamma'_{j,t})$  is a current present value of the



acquirer,  $V(N_{m,j,t}; \Gamma_{j,t})$  is a current present value of target  $m$ ,  $V(N_{i,j,t} + N_{m,j,t}; \Gamma'_{j,t})$  is a present value of the acquirer after purchasing target  $m$ . Each valuation shock is defined by  $\epsilon_m = \tilde{\epsilon}_m - \bar{\epsilon}_0$ , and  $\{\tilde{\epsilon}_m\}_{m \in \{0\} \cup T_{i,j,t}}$  are random variables drawn from Gumbel distribution with scale parameter  $\sigma$  and mean 0. With this specification of valuation shocks, the probability of the acquirer making offer to  $m$  is given by

$$\text{CCP}(m) = \begin{cases} \frac{\exp\{\sigma^{-1}\{V(N_{i,j,t} + N_{m,j,t}; \Gamma'_{j,t}) - V(N_{i,j,t}; \Gamma_{j,t}) - V(N_{m,j,t}; \Gamma_{j,t})\}\}}{\sum_{m \in T_{i,j,t}} \exp\{\sigma^{-1}\{V(N_{i,j,t} + N_{m,j,t}; \Gamma'_{j,t}) - V(N_{i,j,t}; \Gamma_{j,t}) - V(N_{m,j,t}; \Gamma_{j,t})\}\} + 1} & (m \in T_{i,j,t}) \\ \frac{1}{\sum_{m \in T_{i,j,t}} \exp\{\sigma^{-1}\{V(N_{i,j,t} + N_{m,j,t}; \Gamma'_{j,t}) - V(N_{i,j,t}; \Gamma_{j,t}) - V(N_{m,j,t}; \Gamma_{j,t})\}\} + 1} & (m = 0) \end{cases}$$

and ex-ante expected value of a merger opportunity is written by

$$\mathbb{E} \max \left\{ 0, \max_{m \in T_{i,j,t}} \left\{ V(N_{i,j,t} + N_{m,j,t}; \Gamma'_{j,t}) - V(N_{i,j,t}; \Gamma_{j,t}) - V(N_{m,j,t}; \Gamma_{j,t}) + \epsilon_m \right\} \right\} = \sum_{m \in T_{i,j,t}} \text{CCP}(m) \left( V(N_{i,j,t} + N_{m,j,t}; \Gamma'_{j,t}) - V(N_{i,j,t}; \Gamma_{j,t}) - V(N_{m,j,t}; \Gamma_{j,t}) \right) + \sum_{m \in T} \text{CCP}(m) \mathbb{E}[\epsilon_m].$$

Therefore, firm  $i$ 's optimal merger decisions is characterized by  $\{\text{CCP}(m)\}_{m \in T_{i,j,t}}$ .

The role of valuation shocks  $\{\epsilon_m\}$  is primarily to smooth out merger decisions in order to avoid the issue of non existence of value functions of the firm's dynamic problem. (c.f. Gowrisankaran (1999)) Here, the shocks are modeled as transfers from the households. For each target  $m \in T_{i,j,t}$ , the acquirer privately observes subsidies  $\{\epsilon_m\}$  to make a purchase of the target. When the merger is realized, subsidy  $\epsilon_m$  is transferred from the households to the acquirer. <sup>3</sup>

### 3.7 Merger policy

Our study focuses on a simple form of merger policy where any merger that reduce the number of firm in an industry less than  $\underline{N}$  is blocked with probability one. Therefore,  $\underline{N}$  controls the set of merger targets  $T_{j,t}$ . This type of merger policy is used in Igami and Uetake (2019).

### 3.8 Bellman equation

Given the information above, the Bellman equation of a firm in an industry with firm distribution  $\Gamma$  given other firm strategies on production, replication  $\ell^{rd*}(\cdot; \cdot)$ , merger decision  $\text{CCP}^*(\cdot; \cdot, \cdot)$ , and entry intensities  $\varepsilon(\cdot)$  is

<sup>3</sup>Another interpretation is that the acquirer needs to pay transaction cost or fee to brokers. Under this interpretation, the resource constraint of the aggregate consumption needs to be modified as  $C_t = C_t^d + \text{transaction cost}_t$ .

$$\begin{aligned}
(r_t + \lambda)V_t(n; \Gamma) - \dot{V}_t(n; \Gamma) &= \pi_t(n; \Gamma) - w_t f \\
&+ \max_{\ell^{rd}} M(\ell^{rd}, n) [V_t(n+1; \Gamma - \boldsymbol{\iota}_n + \boldsymbol{\iota}_{n+1}) - V_t(n; \Gamma)] - w_t \ell^{rd} \\
&+ \sum_{k=1}^{\infty} k \Gamma_{-n}(k) M_t(\ell^{rd*}(k; \Gamma), k) [V_t(n; \Gamma - \boldsymbol{\iota}_k + \boldsymbol{\iota}_{k+1}) - V_t(n; \Gamma)] \\
&+ \delta n [V_t(n-1; \Gamma - \boldsymbol{\iota}_n + 1\{n \geq 2\}\boldsymbol{\iota}_{n-1}) - V_t(n; \Gamma)] \\
&+ \delta \sum_{k=1}^{\infty} k \Gamma_{-n}(k) [V_t(n; \Gamma - \boldsymbol{\iota}_k + 1\{k \geq 2\}\boldsymbol{\iota}_{k-1}) - V_t(n; \Gamma)] \\
&+ \theta \mathbb{E} \max \left\{ 0, \max_{m \in \Gamma \setminus \{n\}} \{V_t(n+m; \Gamma - \boldsymbol{\iota}_n - \boldsymbol{\iota}_m + \boldsymbol{\iota}_{n+m}) - V_t(n; \Gamma) - V_t(m; \Gamma) + \epsilon_m\} \right\} \\
&+ \theta \sum_{k=1}^{\infty} \Gamma_{-n}(k) \left( \sum_{m=1}^{\infty} \text{CCP}^*(k; m, \Gamma) (V_t(n; \Gamma - \boldsymbol{\iota}_k - \boldsymbol{\iota}_m + \boldsymbol{\iota}_{k+m}) - V_t(n; \Gamma)) \right) \\
&+ \varepsilon(\Gamma) [V_t(n; \Gamma + \boldsymbol{\iota}_1) - V_t(n; \Gamma)].
\end{aligned}$$

Notice that the firm takes into account not only changes to its own state and entry rate as in Klette and Kortum (2004), but also changes to firms in the same industry  $j$ .

### 3.9 Dominant firms and fringe firms

While the setup and the bellman equation is straightforward, we actually solve a simplified version of it. Computationally, keeping track of  $\Gamma$  is difficult because the domain of  $\Gamma$  is  $\mathbb{N}^\infty$ , which means we have to keep track of an infinite dimension object.

To make our setup computationally tractable, modifies the problem in a following way. First, we assume that the first four firms can hold more than one blueprints, conduct replication, and engage in merger and acquisition. We name such firms as dominant firms. Fringe firms can only hold one blueprint and do not have access to replication or merger and acquisition. Dominant firms and fringe firms are subject to different overhead labor. When households attempt to start a new firm, they recognize the new firm will be a dominant firm or a fringe firms. When a dominant firm exits due to blueprint depreciation or merger, a fringe firm is randomly chosen and becomes a dominant firm immediately.

With these assumption, we can rewrite the firm distribution as

$$\Gamma = (n_f, d)$$

where  $n_f \geq 0$  is the number of fringe firms and  $d = (d_1, d_2, d_3, d_4) \in \mathbb{N}^4$  records blueprints of the first four firms, and they must satisfy  $0 \leq d_1 \leq d_2 \leq d_3 \leq d_4$  and  $n_f = 0$  if  $d$  has an element with 0.

### 3.10 The Bellman equation of dominant firms

With some abuse of notation, the bellman equations of dominant firms and fringe firms,  $V$  and  $V^f$ , respectively, are

$$\begin{aligned}
(r + \lambda)V_t(n; \Gamma) - \dot{V}_t(n; \Gamma) &= \pi(n; \Gamma) - w_t f^d \\
&+ \max_{\ell^{rd}} M(\ell^{rd}, n) [V_t(n + 1; \Gamma - \boldsymbol{\iota}_n + \boldsymbol{\iota}_{n+1}) - V_t(n; \Gamma)] - w \ell^{rd} \\
&+ \sum_{k=1}^{\infty} k \Gamma_{-n}(k) M(\ell^{rd*}(k; \Gamma), k) [V_t(n; \Gamma - \boldsymbol{\iota}_k + \boldsymbol{\iota}_{k+1}) - V_t(n; \Gamma)] \\
&+ \delta n [V_t(n - 1; \Gamma - \boldsymbol{\iota}_n + 1\{n \geq 2\}\boldsymbol{\iota}_{n-1}) - V_t(n; \Gamma)] \\
&+ \delta \sum_{k=1}^{\infty} k \Gamma_{-n}(k) [V_t(n; \Gamma - \boldsymbol{\iota}_k + 1\{k \geq 2\}\boldsymbol{\iota}_{k-1}) - V_t(n; \Gamma)] \\
&+ \delta n_f (V_t(n; n_f - 1, d) - V_t(n; n_f, d)) \\
&+ \theta \mathbb{E} \max \left\{ 0, \max_{m \in d \setminus \{n\}} \{V_t(n + m; \Gamma - \boldsymbol{\iota}_n - \boldsymbol{\iota}_m + \boldsymbol{\iota}_{n+m}) - V_t(n; \Gamma) - V_t(m; \Gamma) + \epsilon_{m,t}\} \right\} \\
&+ \theta \sum_{k=1}^{\infty} \Gamma_{-n}(k) \left( \sum_{m=1}^{\infty} \text{CCP}^*(k; m, \Gamma) (V_t(n; \Gamma - \boldsymbol{\iota}_k - \boldsymbol{\iota}_m + \boldsymbol{\iota}_{k+m}) - V_t(n; \Gamma)) \right) \\
&+ \varepsilon(\Gamma) [V_t(n; \Gamma + \boldsymbol{\iota}_1) - V_t(n; \Gamma)]
\end{aligned}$$

and

$$\begin{aligned}
(r + \lambda + \delta)V_t^f(\Gamma) - \dot{V}_t^f(\Gamma) &= \pi(1; \Gamma) - w_t f^f \\
&+ \delta(n_f - 1) \left( V_t^f(n_f - 1, d) - V_t^f(n_f, d) \right) \\
&+ \sum_{k=1}^{\infty} k \Gamma_{-n}(k) M(\ell^{rd*}(k; \Gamma), k) \left[ V_t^f(\Gamma - \boldsymbol{\iota}_k + \boldsymbol{\iota}_{k+1}) - V_t^f(\Gamma) \right] \\
&+ \delta \sum_{k=1}^{\infty} k(\Gamma(k)) \left[ V_t^{f*}(\Gamma - \boldsymbol{\iota}_k + 1\{k \geq 2\}\boldsymbol{\iota}_{k-1}) - V_t^f(\Gamma) \right] \\
&+ \theta \sum_{k=1}^{\infty} \Gamma(k) \left( \sum_{m=1}^{\infty} \text{CCP}^*(k; m, \Gamma) \left( V_t^{f*}(\Gamma - \boldsymbol{\iota}_k - \boldsymbol{\iota}_m + \boldsymbol{\iota}_{k+m}) - V_t^f(\Gamma) \right) \right) \\
&+ \varepsilon(\Gamma) \left[ V_t^f(n_f + 1, d) - V_t^f(\Gamma) \right]
\end{aligned}$$

where  $V_t^{f*}(\cdot)$  is an expected value of a fringe firm before one of the fringe firms is randomly chosen to be a new dominant firm,

$$V_t^{f*}(\Gamma') = \begin{cases} \frac{1}{n_f} V_t(1; \Gamma') + \left(1 - \frac{1}{n_f}\right) V_t^f(\Gamma') & \text{if a dominant exits from } \Gamma \text{ to } \Gamma' \\ V_t^f(\Gamma') & \text{otherwise.} \end{cases}$$

Here,  $\iota_n$  is abuse of notation since it now conceptually represents change the distribution of dominant firms.

### 3.11 Households optimization

Households choose non-negative values  $\{C_t^d, a_t, L_t, \{\ell_{t,j}^e\}_{j \in [0,1]}\}_{t \geq 0}$  to maximize

$$\int_0^\infty e^{-\rho t} \ln C_t^d dt$$

subject to

$$\begin{aligned} \int_0^\infty \exp\left(-\int_0^s r_u du\right) P_s C_s^d ds &\leq a_0 \\ &+ \int_0^\infty \exp\left(-\int_0^s r_u du\right) \left( w_s L_s + \int_0^1 E(\ell_{j,s}^e) V_s(1; \Gamma_{j,s} + \iota_1) dj - \theta \int_0^1 \sum_{k \in d_{j,t}} \mathbb{E}[\epsilon_{j,k,m,t}] dj \right) ds \\ L_t + \int_0^1 \ell_{j,t}^e dj &\leq 1 \quad (\forall t) \\ \liminf_{T \rightarrow \infty} \exp\left(-\int_0^T r_s ds\right) a_T &\geq 0 \\ &\text{given } a_0, \{P_t, w_t, r_t, \{\{V_t(1; \Gamma_{j,t} + \iota_1), \int_0^1 \sum_{k \in d_{j,t}} \mathbb{E}[\epsilon_{j,k,m,t}] dj\}_{j \in [0,1]}\}_{t \geq 0}\} \end{aligned}$$

where  $\int_0^1 \sum_{k \in d_{j,t}} \mathbb{E}[\epsilon_{j,k,m,t}] dj$  is the expected valuation transfer from households to firms, and the expectation is taken over the conditional probability choice of acquisition targets.

The intertemporal condition reads the interest rate is equal to

$$r_t = \rho + \frac{1}{P_t C_t^d} \frac{d(P_t C_t^d)}{dt}.$$

Finally, the entry effort of industry  $j$ ,  $\ell_{j,t}^e$  is pinned down by the first order condition with respect to  $\ell_{j,t}^e$ ,

$$w_t = E'(\ell_{j,t}^e)V_t(1; \Gamma_{j,t} + \boldsymbol{\nu}_1) \text{ if } V_t(1; \Gamma_{j,t} + \boldsymbol{\nu}_1) > 0$$

and otherwise  $\ell_{j,t}^e = 0$ . Therefore,  $e_{j,t} = E(\ell_{j,t}^e)$  with optimal values for  $\ell_{j,t}^e$ . This pins down the entry rate of industry  $j$  with the firm distribution  $\Gamma_{j,t}$ ,  $\varepsilon(\Gamma_{j,t})$ .

### 3.12 Equilibrium

An equilibrium of the economy satisfies

1. Household optimize given prices, values of new firms, and valuation shocks
2. Each industry  $j \in [0, 1]$  is in a Markov perfect equilibrium given aggregate prices, quantities and entry rates.
3. Market clearing for all  $t$ ,

(a) Labor market

$$1 - \int_0^1 \ell_j^e dj = \int_0^1 \left( \sum_{i \in I_j} (\ell_{ji}^{\text{pd}} + \ell_{ji}^{\text{rd}}) \right) dj$$

(b) Final goods market

$$C_t^d = C_t$$

(c) Asset market

$$a_t = \int_0^1 \left( \sum_{i \in I_j} V_{ji} \right) dj.$$

### 3.13 Steady state

We focus on a steady state equilibrium of the economy. A steady state equilibrium of this economy satisfy industry states  $\{\Gamma_j\}$ , industry entry efforts  $\{\ell_j^e\}$ , aggregate consumption level  $C_t^d = C_t = C$ , aggregate price level  $P_t = P$ , nominal wage  $w_t = w$ , interest rate  $r_t = \rho$ .

## 4 Examples of Lifecycle of Industries

This section describes the lifecycle patterns the model aims to capture in the model with some example. Gort and Klepper (1982) and Klepper and Graddy (1990) collected data of multiple industries and found that a

typical pattern is found over the lifecycle of the industries. They also find there is a non-monotonic pattern of the number of firms in the early stage of their lifecycle: the number of firms builds up gradually, and then it drops for relatively short period, and much fewer firms survives in the long run.

#### 4.1 U.S. automobile tire industry

An example of the U.S. automobile industry is drawn from Jovanovic and MacDonald (1994). The U.S. automobile industry 1906-73. The stylized facts of industry dynamics is summarized by them “a young industry is populated by a few small firms, and the products commands a high price. Entry then expands the number of firms and each produces more, the combined effect being to raise output dramatically and lower price. Output growth persists, but the rate of growth falls below the growth rate of average firm size so that the firms must exit – a “shakeout.” ”

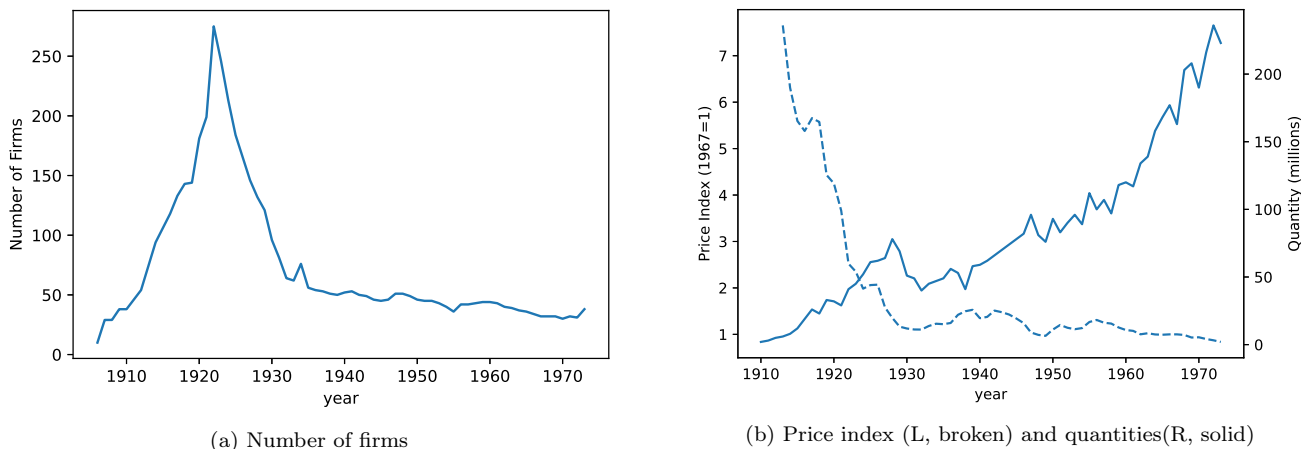


Figure 1: Lifecycle of the U.S. automobile tire industry

Source: Jovanovic and MacDonald (1994)

Figure 1 exemplify lifecycle of industries which the our model captures. An industry starts with a few firms, and their product command high prices. Next, entry expands the number of firms, quantity rises, and price drops. The number of firms decline dramatically but outputs growth persists – a lifecycle stage which is observed in many industries<sup>4</sup>. In the long run, fewer numbers of firms survive.

<sup>4</sup>The lifecycle patter characterized by the shakeout has been observed in many industries. For example,

## 4.2 World wide hard disk industry

Another example is taken from Igami and Uetake (2019) who study merger policy using data from the hard disk drive industry.<sup>5</sup> Their data show that the industry experienced the typical non-monotonic pattern of the number of firms.

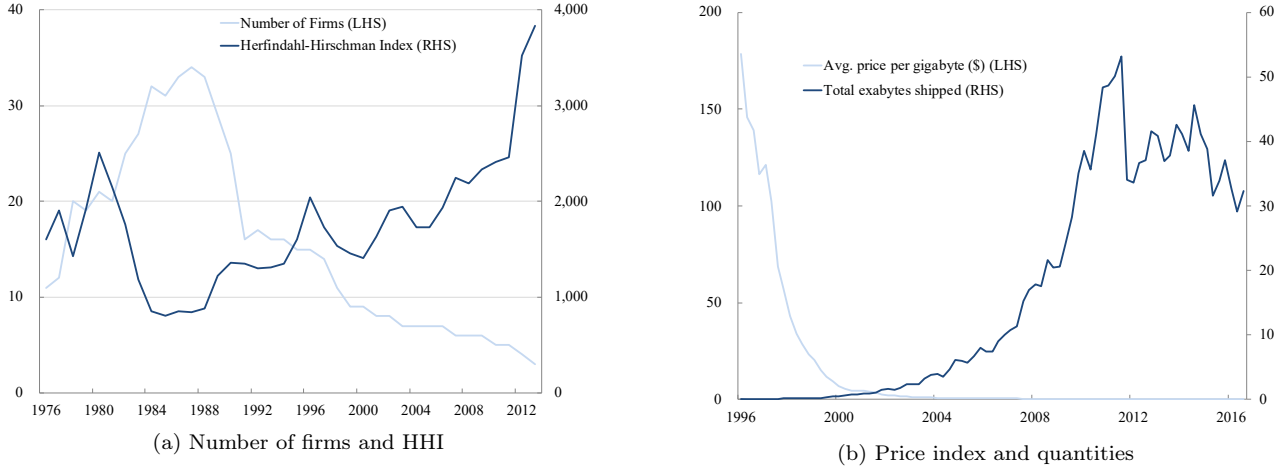


Figure 2: Lifecycle of the hard disk drive industry (1)

Source: Igami and Uetake (2019)

Figure 2 show time paths of number of firms, market concentration measured by HHI, unit price of hard disk drive, and shipment of hard disk drive in unit of exabytes. As in the U.S. automobile tire industry, the path of the number of firm shows non-monotonic pattern over the lifecycle. In line with the number of firms, we also confirm the non-monotonic path of market concentration.

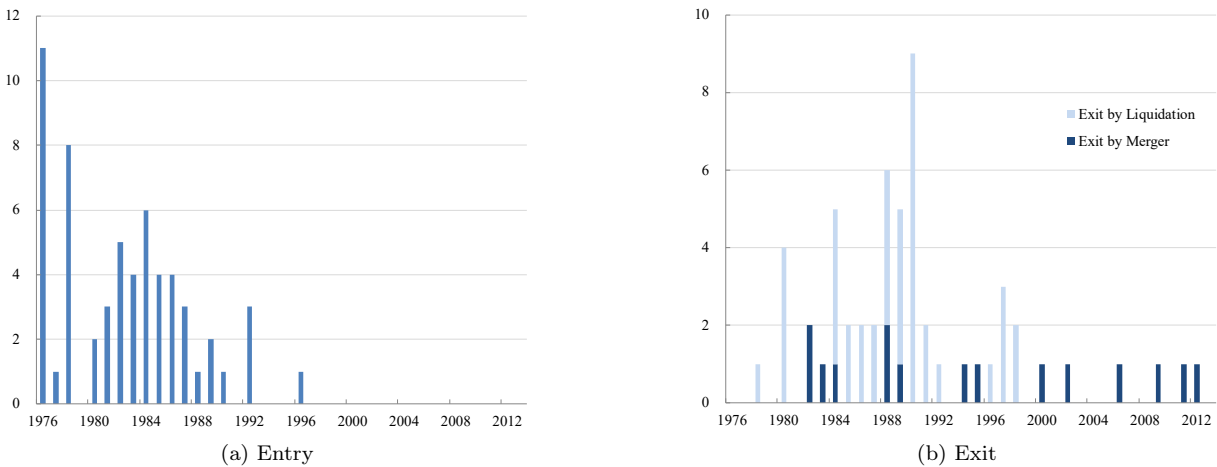


Figure 3: Lifecycle of the hard disk drive industry (2)

Source: Igami and Uetake (2019)

<sup>5</sup>We thank Mitsuru Igami and Kosuke Uetake for providing the data.

Data from the hard disk drive industry also contains entry and exit behavior of the industry in Figure 3. First, we see entry is active in the early stage of its lifecycle, and the number of entry gradually dies out over time. Second, the exit of firms are peaked around the time of the shakeout from the late 1980s to the early 1990s. This is consistent with a typical of shakeouts where a shakeout is explained by both decline in entry and increase in exits.

## 5 Parameter Choice and model behavior

This section describe the preliminary choice of parameters in the benchmark economy. The parameters are chosen so that the model generates the salient features of industry lifecycles described in Section 4.

Table 1: Benchmark Parameterization

	Notation	Value	Explanation
Time discount rate	$\rho$	0.04	Time unit is year
Periodic utility	$\log c$		
Labor supply		1	Inelastic
Cross industry elasticity	$\eta$	2	To generate long-run industry growth
Within industry elasticity	$\xi$	5	To generate long-run industry growth
Arrival rate of merger opportunity	$\theta$	0.25	A merger opportunity arrives every year when 4 dominant firms are active
Scale parameters of valuation shock	$\sigma$	0.05	Large enough so that the VFI converges
Overhead labor cost of fringe firms	$f^f$	0	To generate many entries in the beginning
Overhead labor cost of dominant firms	$f^d$	0.04	To match aggregate overhead labor ratio
Blueprint depreciation rate	$\delta$	0.10	Capital depreciation rate
Obsolete shock intensity	$\lambda$	0.02	Population growth of 2%
Entry tech. $E = e_0 \ell^{e_1}$	$e_0$	20	To generate many entries in the beginning
	$e_1$	0.8	high entry elasticity
Replication tech. $\mu(\ell) = m_0 \ell^{m_1}$	$m_0$	0.45	To generate slower growth of dominant firms in the beginning and long-run industry growth
	$m_1$	0.23	
Productivity level	$z$	1	Normalization
Threshold for merger block policy	$\underline{N}$	3	Igami and Uetake (2020)



## 5.1 Preference

Long run industry growth hinges on values of both  $\eta$ , cross-industry elasticity and  $\xi$ , within-industry elasticity. Specifically, the variety effect disappear if  $\eta$  is close to one or  $\xi$  is very large<sup>6</sup>. If  $\eta$  is close to one, expenditure shares on each industry commodity is constant and households do not enjoy variety from industry commodity. If  $\xi$  is very large, within-industry commodities from distinct blueprints get closer to homogeneous good and variety effect from multiple blueprints again disappears.

The other extreme can be achieved by setting  $\eta = \xi$ , with which firm growth exhibit Gibrat's law as in Klette and Kortum (2004). We choose these parameters so that the model does not generate Gibrat's law because the example from the hard disk drive industry shows that the industry output growth stops in the long horizon. Luttmer (2011) also finds growth rates of very large firms tend to decline as they get older. Given these observations, we choose  $\eta = 2$  and  $\xi = 5$  as such an example.

## 5.2 Entry technology

The lifecycle pattern also hinges on the parameters of the entry technology  $E(\ell) = e_0(\ell)^{e_1}$ .  $e_0$  captures the efficiency of the entry technology and  $e_1$  is the labor share of the entry technology. Given the first order condition  $w = E'(\ell_{j,t}^e)V_j^e(\Gamma_j) = e_0e_1(\ell)^{e_1-1}$ ,  $e_1$  controls entry intensity elasticity with respect to the ratio of the entry value  $V_j^e(\Gamma_j)$  to wage  $w$ .  $e_0$  controls the efficiency of the entry technology. We find that the two parameters shape shakeouts.  $e_1$  controls how quickly the number of firm drops during shakeouts while  $e_0$  controls how large the overshoot of the number of firms during shakeouts. Given that shakeouts happens fairly short periods of time (say 10 to 20 years) and the size of overshoot is large in the example, we set the labor share parameter  $e_1$  large (0.8) and the efficiency parameter  $e_0$  also large (20) to roughly match shakeouts in the model to the one in the hard disk drive industry.

## 5.3 Replication technology

The replication technology is also assumed to be Cobb-Douglas with a functional form  $\mu(\ell) = m_0(\ell)^{m_1}$ , where  $m_0$  captures the efficiency of the blueprint replication technology and  $m_1$  is the labor share of it. We find that path of industry output  $\{y_{j,a}\}$  exhibits longer persistent growth if the labor share  $m_1$  is smaller and the

---

<sup>6</sup>To see this point, think of aggregate output in an environment where each industry  $j$  is controlled by a monopolist and is characterized by the number of blueprints  $N_{j,t}$ . In this case, the aggregate output is

$$C_t = \left( \int_0^1 N_{j,t}^{\frac{\eta-1}{\xi-1}} dj \right)^{\frac{1}{\eta-1}}.$$

$\{N_{j,t}\}$  becomes irrelevant to  $C_t$  when  $\eta \rightarrow 1$  or  $\xi \rightarrow \infty$ .

industry output path quickly converges without shakeouts when  $m_1$  is closer to 1. We also find the industry lifecycle does not experience shakeouts if the efficiency parameter  $m_0$  is large because dominant firms quickly accumulate blueprints. Therefore,  $m_0$  and  $m_1$  are chosen to generate persistent growth of industry output and shakeouts in the calibration.

## 5.4 Blueprint depreciation and industry obsolescence

We set the blueprint depreciation rate  $\delta = 0.10$ , slightly higher than depreciation rate of physical capital. We set it so because firms with negative present value cannot stop operation by themselves and we are concerned that firms with negative present values keep operating too long<sup>7</sup>.

The Poisson intensity of obsolescence shock  $\lambda$  is set to 0.02 to get the population growth rate of 2%<sup>8</sup>. Although our economy does not feature economic growth, we can think of it as a semi-endogenous growth model with a mass of population growing at a rate  $\lambda$  and entrepreneurs discover industry specific fixed assets endogenously à la Roy model<sup>9</sup>. It is tempting to match industry age moments from the model to those from data. Given that the age distribution of industries at the steady state is an exponential distribution with the parameter  $\lambda$ , the mean and the median of industry age are  $1/\lambda$  and  $(\ln 2)/\lambda$ , respectively. At this moment, we are ignorant of informative data source to directly pin down these moments. With  $\lambda = 0.02$ , the predicted mean and median of industry ages at the steady state is  $1/\lambda = 50$  years and  $\ln 2/\lambda \sim 34.7$  years, respectively<sup>10</sup>.

## 5.5 Overhead labor

Firm level overhead labor captures economy of scale. It is often the case that an antitrust agency reviewing a merger case faces a tradeoff between higher market power and reducing some type of technological efficiency such as reducing marginal cost or saving overhead cost. Introducing firm level overhead labor is a simple way to capture technological efficiency achieved in mergers.

We find that the overhead cost of fringe firms strongly discourages entries and it is difficult to generate overshoot in the firm number. It is especially true if set  $f^f = f^d > 0$ . To take the other extreme, we set

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<sup>7</sup>The assumption that firms are not allowed to choose to exit is for simplification. In principle, one can introduce exit decisions in the model by introducing smoothness in exit decisions. (For example, Fudenberg and Tirole (1986) use private information of a firm's outside option value for smoothing.)

<sup>8</sup>In the next calibration we plan to set  $\lambda = 0.01$  since it is much closer to the U.S. population growth rate.

<sup>9</sup>Luttmer and Yao (2020) study an economy with endogenous creation of industry specific fixed assets and population growth.

<sup>10</sup>One may want to relate these moments of industry age to the median age of large firms in the data. For example, Luttmer (2011) points out the median age of firms with more than 10,000 employees in 2008 was about 75 years. Since dominant firms rarely die due to blueprint depreciation, age of industries is closely related to age of dominant firms. The predicted value by model is much lower than 75 years. Firms in our model, however, operate only in one industries and therefore tends to be short-lived compared to large firms operating in multiple industries in the data.

$f^f = 0$ . We can interpret that the difference between  $f^f$  and  $f^d$  reflects the fact that fringe firms do not have access to blueprint replication technology or merger opportunity.  $f^d$  is set to 0.04 and the resulting overhead labor ratio – the ratio of aggregate overhead labor to the labor supply – is about 13.5%. Ramey (1991) concludes 20% is a reasonable estimate of the overhead labor ratio in the U.S. manufacturing sector. Given that we are taking an extreme case  $f^f = 0$ , the model implied overhead ratio of 13.5% is not too off from the estimate.

## 5.6 Merger relevant parameters

We choose two parameters  $(\theta, \underline{N})$  so that relevant moments fall in the magnitude of the hard disk drive industry described by Igami and Uetake (2019).

In our model, merger market is under friction and firms can engage in blueprint transactions infrequently. Each dominant firm receives an opportunity to acquire another dominant at the rate  $\theta$ . We pick  $\theta = 0.25$  so that a merger opportunity arrives in an industry every year when four dominant firms are operating in an industry. In the hard disk drive industry, mergers are observed at most twice in a year.

We focus on a simple merger policy that mergers will be blocked if the number of incumbents is less than or equal to  $\underline{N}$ . We choose  $\underline{N} = 3$  as a baseline policy following Igami and Uetake (2019)<sup>11</sup>.

The scale parameter of valuation shock  $\sigma$  is remain to be chosen. Since the main purpose of introducing the valuation shock is to achieve a fixed point,  $\sigma$  is currently chosen so that it is large enough that value function iteration converges<sup>12</sup>.

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<sup>11</sup>Igami and Uetake (2019) find  $\underline{N} = 3$  is a reasonable description of the actual policy in high-tech markets between 1996 and 2011.

<sup>12</sup>Gowrisankaran (1999) points out endogenous merger decisions without smoothing end in non-existence of a fixed point in firms' dynamic problem. Our formulation of mergers is close to Jeziorski (2015), where acquirers are subject to random components of transaction cost.

## 5.7 Model implied industry dynamics

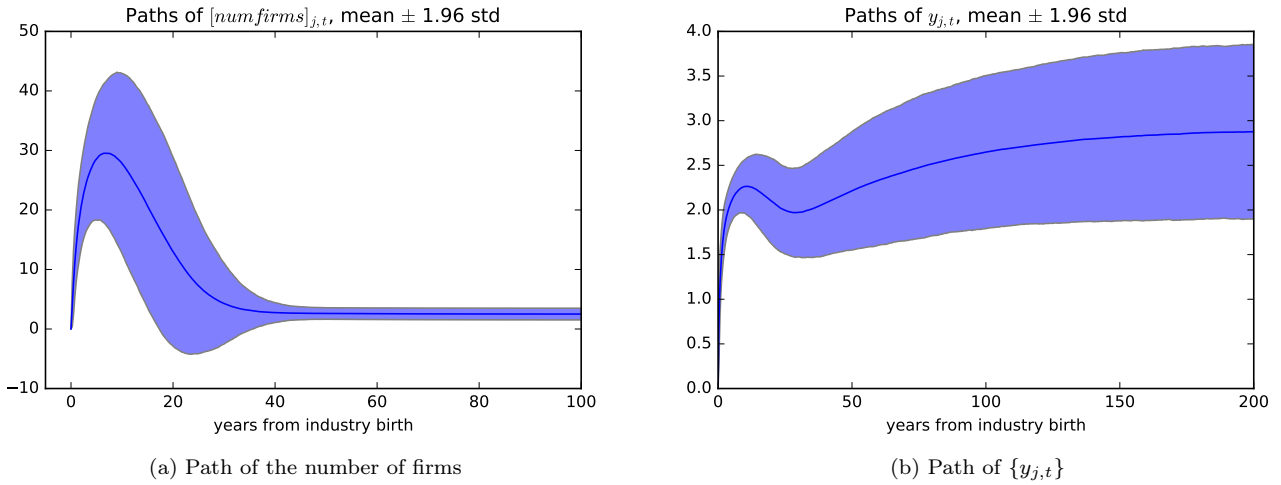


Figure 4: Lifecycle of industries in the model (1)

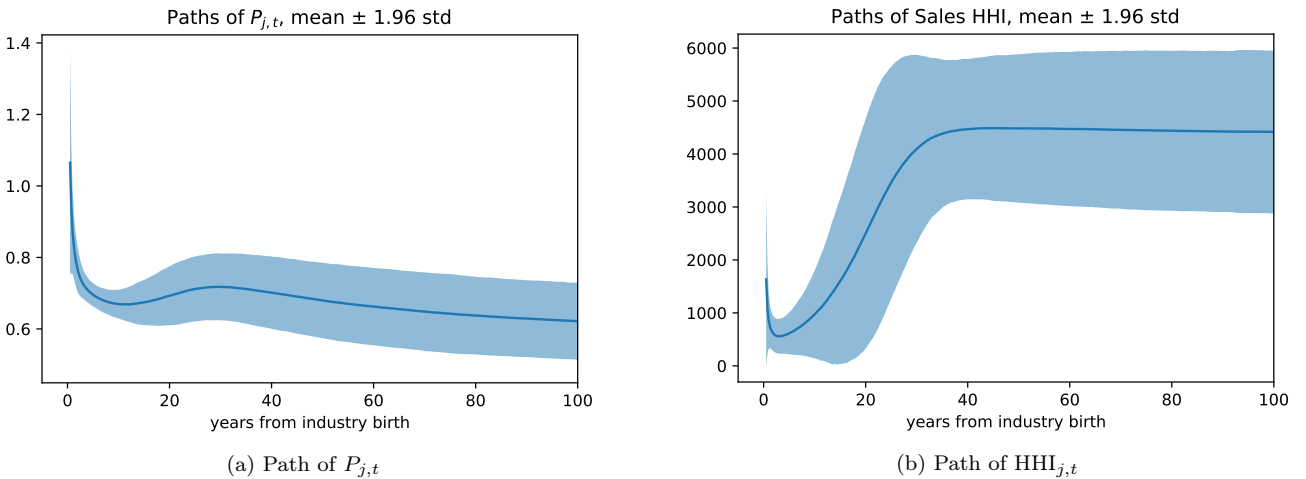


Figure 5: Lifecycle of industries in the model

Figure 4 and Figure 5 shows the means of the lifecycle pattern across industries. Each time series is a time path of cross sectional mean of the moment, and period-by-period cross-sectional confidence intervals are also plotted. While it is true each industry exhibit different lifecycle paths due to randomness, these figures show the common pattern of the lifecycle across industry.

According to Figure 4 and Figure 5, the model captures the non-monotonic time paths of the number of firms and the market concentration index. While the output path and price index path shows the long-run trends as in the data, the output path converges very quickly compared to the examples of the quantity in

Section 4.

## 6 Industry view of merger policy

In this section, we study how merger policy affects industry outcomes in an economy with the parameters we pick in the previous section.

### 6.1 Welfare criterion in industry analysis

Industry analysis of antitrust policy often uses consumer surplus to evaluate the policy<sup>13</sup>. Here, we define static and present-value consumer surplus of an industry which resemble the welfare criteria industry analyses use before studying merger policy from the industry perspective. Remember industry  $j$  faces an inverse demand curve

$$\frac{p_{j,t}}{P_t} = \left( \frac{C_{j,t}}{C_t} \right)^{-\frac{1}{\eta}}.$$

Assume that  $PC^{\frac{1}{\eta}} = 1$  for normalization at the steady state, the demand curve for industry  $j$  is simply a function of industry- $j$  price index  $p_j$ ,

$$y(p_j) = p_j^{-\eta}.$$

We define static consumer surplus of industry  $j$  with industry price index  $p_j$  and quantity  $C_j$  as an upper triangle in the price-quantity diagram,

$$\begin{aligned} \text{CS}_j(C_j) &= \int_0^{C_j} (p(y) - p(C_j)) dy \\ &= \frac{1}{\eta - 1} C_j^{\frac{\eta-1}{\eta}}. \end{aligned}$$

We define expected present value consumer surplus of an industry at state  $\Gamma$ ,  $\text{pCS}(\Gamma)$ , by

$$\text{pCS}(\Gamma) = \mathbb{E} \left[ \int_0^{\infty} e^{-(r+\lambda)a} \text{CS}(C(\Gamma)) da | \Gamma \right].$$

where  $C(\cdot)$  is industry output when the industry output is  $\Gamma$  and the expectation is taken over the randomness of industry lifecycle. The future consumer surpluses are discounted by  $r + \lambda$ , not  $r$ , to take into account the

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<sup>13</sup>For example, Igami and Uetake (2019) evaluate merger policies using present value consumer surplus, producer surplus, and total surplus.

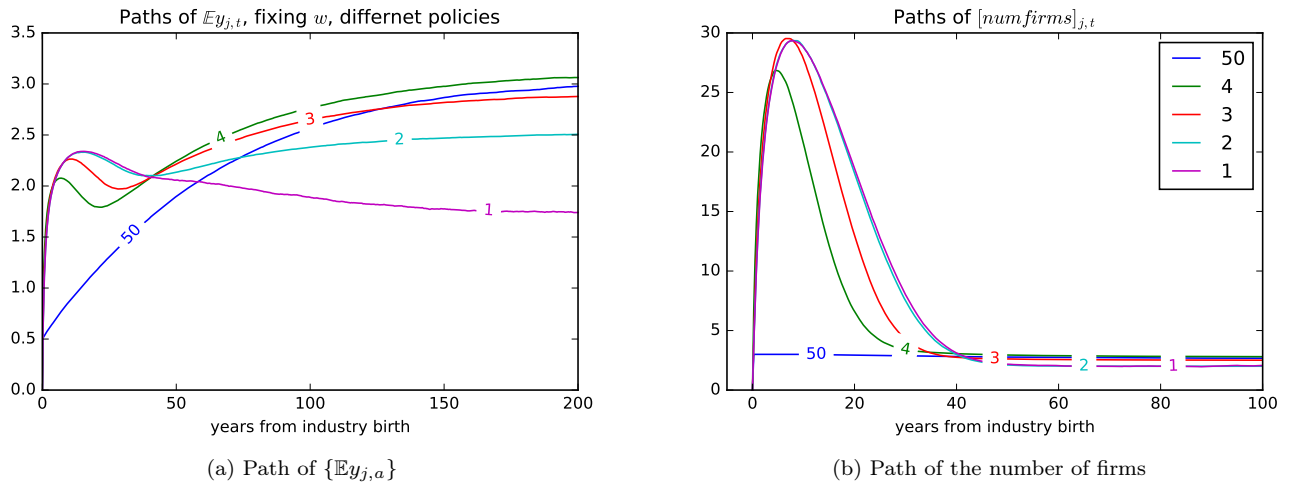


Figure 6: Differential effects of merger policy

fact that the industry will be obsolete with the arrival rate  $\lambda$ . We can also define the mean of present value consumer surplus at the age  $a \geq 0$ ,  $pCS_a$ , by

$$pCS_a = \mathbb{E} \left[ \mathbb{E} \left[ \int_0^\infty e^{-(r+\lambda)t} CS(C(\Gamma)) dt | \Gamma_a \right] \right] \quad (2)$$

where the outer expectation is taken over the steady state distribution of  $\Gamma$  at the age  $a$ . Since all the industry start with no firms, the age-0 present value consumer surplus of any industry is  $pCS_0$  with  $\Gamma_0 = (0, 0, 0, 0, 0)$ .

Finally, we note that steady state consumption  $C$  can be expressed in terms of static consumer surplus  $CS_j$ , by

$$C = \left[ \int_0^1 (\eta - 1) CS_j dj \right]^{\frac{\eta}{\eta-1}}.$$

## 6.2 Effect of merger policy at over the lifecycle

Now we examine how change in merger policy affects industry dynamics. Here, the merger policy is set to  $\underline{N} = 3$  following Igami and Uetake (2019). Setting  $\underline{N} = 3$  means that the government commits to block a merger when the number of firm in an industry is less than equal to 3. In the experiment, we change  $\underline{N}$  in an infinitesimally small industry keeping  $\underline{N} = 3$  in the other industries. When implementing, we simply fix the aggregate prices and quantities to ignore general equilibrium effect.

Figure 6 shows how industry lifecycle pattern changes over different merger policies  $\underline{N}$  when wage rate is fixed to the one in the benchmark economy. In the left panel, we see that changes in  $\underline{N}$  have two effects on the path of industry output  $\{y_{j,a}\}$ . First, looser merger regulations with  $\underline{N} = 1, 2$  reduce outputs on average

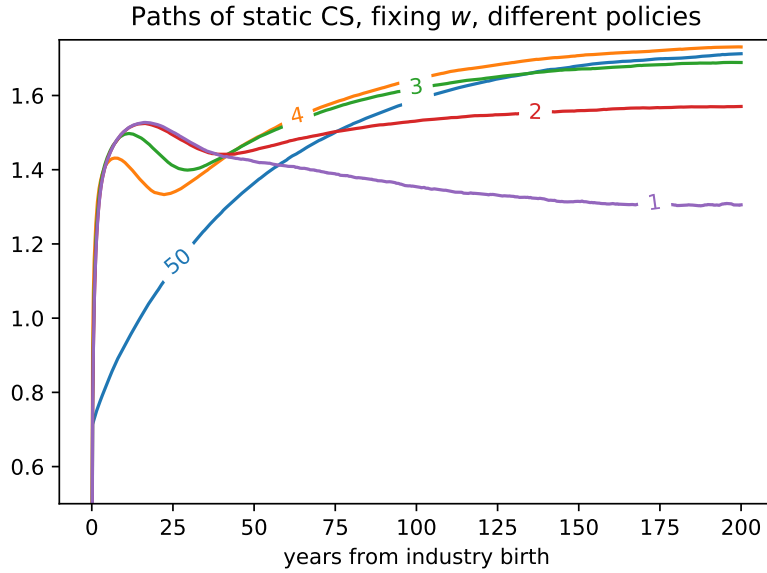


Figure 7: Mean static consumer surplus at different age  $a$

in the relatively long run. Second, the looser merger regulations increase outputs when the industry is young. We see the latter effect strongly when we set  $\underline{N} = 50$ . Compared to  $\underline{N} = 3$ , the output when the industry is young declines and the non-monotonicity of the output path disappears.

The right panel shows how the merger policy affects the time path of the number of firms. We see that the non-monotonicity of the number of firms attenuated as  $\underline{N}$  gets smaller.

Figure 7 shows how static consumer surplus changes over  $\underline{N}$ . Note that this is a simple transformation of the left panel of Figure 6.

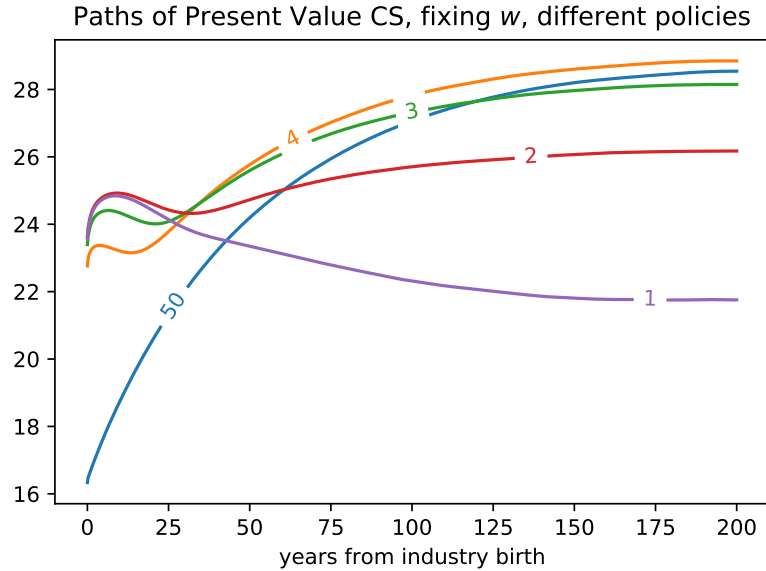


Figure 8: Mean present value consumer surplus at different age  $a$  across merger policies  $\underline{N}$

Given that our environment feature dynamics of firms and industries over time, it is natural to use present values of welfare criterion such as consumer surplus.<sup>14</sup> Figure 8 plots paths of present value consumer surplus defined as (2) over different  $\underline{N}$ .

The findings are straightforward given that Figure 8 is present value transformation of Figure 7. When an industry is young, lenient merger policy achieves higher consumer surplus and tough merger policy like  $\underline{N} = 50$  reduces consumer surplus significantly. When an industry is old enough, lenient merger policy reduces present value consumer surplus due to market power. going to duopoly or monopoly is much worse than going to triopoly.

<sup>14</sup>This is case in many studies such as Berry and Pakes (1993), Mermelstein, Nocke, Satterthwaite and Whinston (2019), and Igami and Uetake (2019)



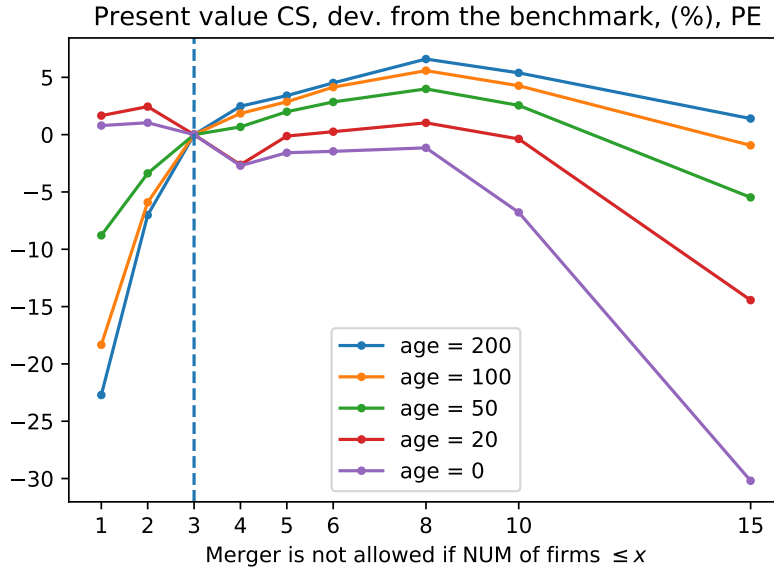


Figure 9: Present value consumer surplus at different age  $a$  across merger policies  $\underline{N}$

Figure 9 plots the same data with  $\underline{N}$  on the x-axis, plotted in terms of percent deviation. Starting from the baseline case  $\underline{N} = 3$ , the gains from changing  $\underline{N}$  is very small in either direction. Especially, loosening the policy to  $\underline{N} = 1$  marginally increase the consumer surplus, and slightly reduces the consumer surplus. If  $\underline{N}$  is loosen to 15, the present value consumer surplus declines by roughly 30%. Therefore, if an antitrust agency's objective is to maximize the expected present value of consumer surplus at the very beginning of the industry,  $\underline{N} = 3$  almost optimal.

We would draw different conclusion if the evaluation was conducted at an later stage of its lifecycle. In the extreme case, if  $a = 200$  years, moving  $\underline{N} = 8$  from 3 increases the present value consumer surplus by around 6%, and  $\underline{N} = 15$  does not reduce the surplus as large as the case with  $a = 0$  year. It would be also the case moving to  $\underline{N} = 1$  will reduce the present value consumer surplus by more than 20%.

## 7 Aggregate Implications of Merger Policy

Section 6 studied merger policy from an industry perspective. This section contrasts it with the planner's perspective. Since the planner can affect all the industries, she ends in a different conclusion from an industry regulator's one in general.

## 7.1 Aggregate welfare criterion

We simply assume that the planner maximizes the steady state aggregate output. Note that aggregate output can be expressed in terms of consumer surplus,

$$\begin{aligned} C_t &= \left[ \int_0^1 C_{j,t}^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \\ &= \left[ \int_0^1 (\eta-1) \text{CS}_{j,t} dj \right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

This means that maximizing the steady state output  $C$  is equivalent to maximizing the sum of consumer surplus  $\int_0^1 \text{CS}_j dj$  at the steady state. The sum is equal to

$$\begin{aligned} \int_0^1 \text{CS}_j dj &= \mathbb{E} [\text{CS}(\Gamma)] \\ &= \int \lambda e^{-\lambda a} \mathbb{E} [\text{CS}(\Gamma_a)] da \end{aligned} \quad (3)$$

where the first expectation is taken over the steady state distribution of  $\Gamma$ , the second one is taken over the steady state distribution of  $\Gamma_a$  conditional on industry age  $a$ , and  $\lambda e^{-\lambda a}$  is the density of industry age at the steady state. Therefore, maximizing the steady state aggregate output is equivalent to maximizing the expected present value consumer surplus with discount rate  $\lambda$ .

On the other hand, an industry regulator chooses  $\underline{N}$  to maximize the present value consumer surplus given the current state of the industry  $\Gamma$  and assuming that the economy is at the benchmark. Since each industry infinitesimally small, change in merger policy in one industry does not change aggregate outcome. The present value consumer surplus given the current industry state  $\Gamma$  is

$$\text{pCS}(\Gamma) = \mathbb{E} \left[ \int_0^\infty e^{-(r+\lambda)a} \text{CS}(C(\Gamma_a)) da | \Gamma_0 = \Gamma \right].$$

This depends on the current industry state  $\Gamma$ , which needs to be specified for us. We avoid this issue by taking an expectation of it over the steady state distribution of industry state given industry age  $a$ ,  $\Gamma_a$ ,

$$\text{pCS}_a = \mathbb{E} \left[ \mathbb{E} \left[ \int_a^\infty e^{-(r+\lambda)(t-a)} \text{CS}(C(\Gamma_{t+a})) dt | \Gamma_a \right] \right] \quad (4)$$

Since we assume that all new industries start with no firm and no blueprint, we can also obtain age-0 present

value of consumer surplus,

$$\begin{aligned} \text{pCS}_0 &= \mathbb{E} \left[ \int_0^\infty e^{-(r+\lambda)t} \text{CS}(C(\Gamma_t)) dt \right] \\ &= \int_0^\infty e^{-(r+\lambda)t} \mathbb{E} [\text{CS}(C(\Gamma_t))] dt \end{aligned} \quad (5)$$

where  $\Gamma_0 = (0, 0, 0, 0, 0)$ .

Lastly, we also defined a  $C$ -equivalent measure of expected present consumer surplus  $\text{pCS}_a$ ,  $\text{pCS}_a^*$ , by

$$\text{pCS}_a^* = \left( \mathbb{E} \left[ \mathbb{E} \left[ \int_a^\infty (\eta - 1)(r + \lambda) e^{-(r+\lambda)(t-a)} \text{CS}(C(\Gamma_{t+a})) dt | \Gamma_a \right] \right] \right)^{\frac{\eta}{\eta-1}}.$$

## 7.2 Discrepancy between the planner's objective and an industry regulator's objective

Comparing (3), (4) and (5) clarify potential discrepancy between the planner's perspective and an industry regulation perspective.

1. discounting: the planner maximizes with discounting  $\lambda$ , but an industry regulator uses
2. general equilibrium
3. old industry bias

The first difference is discount factors that planner and an industry regulator uses. The planner faces industries whose ages are distributed with exponential density  $\lambda e^{-\lambda a}$ . Virtually, this means that the planner discount future profits with the rate of  $\lambda$ . On the other hand, an industry regulator discount rate  $r + \lambda$  since  $r$  is an interest rate and the industry disappears with rate  $\lambda$ .

The second difference is that an industry regulator ignores general equilibrium effect whereas the planner takes the effect into account. For example, looser merger regulation increases production and entry effort in younger industries whereas reduces output in older industries due to higher market power. When the planner changes merger policy  $\underline{N}$  in all the industries, the steady state wage  $w$  needs to be adjusted to clear the labor market.

The third difference is that industry analysis can be biased towards older industries with  $a > 0$ . The fact that a lifecycle aspect of industries is not considered in the existing works means the existing industry analyses implicitly assume they evaluate merger policy when the industry is sufficiently old, or  $a$  is large.

Through the lens of our model, it would be inappropriate to conduct merger analyses at a given industry state  $\Gamma$ , but they should target hypothetical welfare with age 0 welfare criterion.

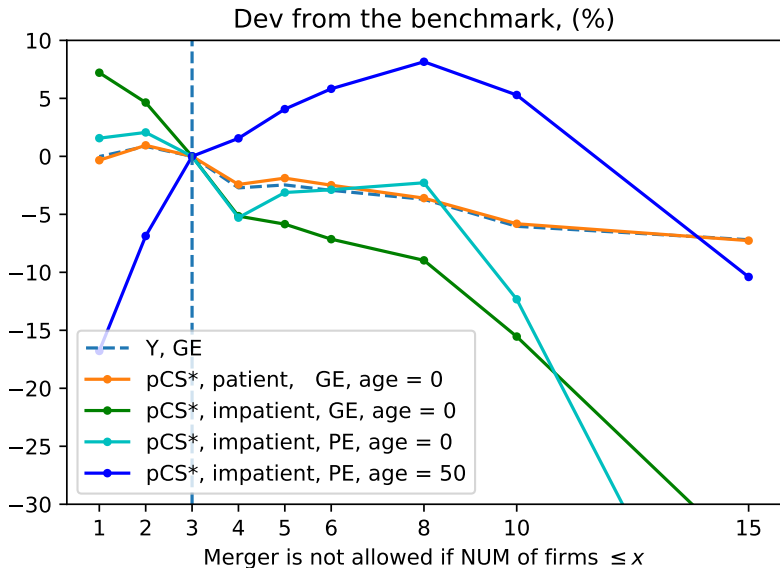


Figure 10: Present value consumer surplus at different age  $a$  across merger policies  $\underline{N}$

Figure 10 visually exemplifies the discrepancy between the two perspectives. Starting from the benchmark economy, the planner's changing  $\underline{N}$  affects the steady state aggregate output  $Y$  moderately. Introducing tougher merger policy with higher  $\underline{N}$  lower the output. If the planner used discount rate  $r + \lambda$  instead of  $\lambda$ , then the implications of merger policy would be slightly different. According to Figure 10, loosing  $\underline{N}$  (or smaller) would be more preferable than it is when the planner the discount rate  $\lambda$ . It is because using larger discount rate  $r + \lambda$  put more emphasis on younger industries. With looser merger policy (smaller  $\underline{N}$ ), there would be more entries at the beginning of the lifecycle at the cost of output drop in the long run. With higher discount rate, the planner cares less about the output loss in the long run. The same logic applies to tighter merger regulation  $\underline{N}$ . Removing the general equilibrium effect overall undoes the impatient effect around  $\underline{N} = 3$ .

Compared to the first two discrepancy, introducing the bias towards older industry bias may affect evaluation of merger policy. According to Figure 10, introducing large old industry bias modify the property of merger policy evaluation. Especially, looser merger policy (smaller  $\underline{N}$ ) now strongly reduces present value consumer surplus, and introducing tighter policy (larger  $\underline{N}$ ) increases the surplus. The old industry bias practically puts more weight on old industries and remove young industries from considerations. Therefore,

the positive effect of looser merger policy that increases industry output when the industry is young will be ignored and an industry regulator faces the negative side of loose merger policy. Overall, with the current parameterization and the strong old industry bias with age 50 years, tighter merger policy criterion with  $\underline{N} = 8$  increases the present value consumer surplus in an industry analysis. From the planner's perspective, it reduced the steady state output compared to  $\underline{N} = 3$ .

## 8 Conclusion

While design of merger policy has been one of the central issues, the literature has been silent on the interaction of merger policy and industry lifecycle. To highlight the relationship, we build a multi-industry macroeconomic model where each industry experiences the typical pattern of the lifecycle of industries.

We find merger policy affects the pattern of industry lifecycles. Especially, paths of the number of firms and outputs in an industry depends on the merger policy – if the policy is tight, it is less likely to have the overshoot in the number of firms as we see in the typical lifecycle of industries. This highlights the entry margin at early stages of the lifecycle.

From the planner's perspective, outputs from industries in early stages of the lifecycle counts. Since industries located at various life stages coexist in an economy, the planner cares about outputs both in younger industries and older industries. On the other hand, an industry regulator who maximizes the present value consumer surplus for the rest of the lifecycle may implicitly ignoring early stages of the lifecycle. Our study underlines the importance of thinking of industry merger policy through the entire lifecycle, not at a stage when an industry is matured and excessive market power becomes a concern.

Our study clarifies a point that design of merger policy should take into account how policies affect an industry lifecycle over its long lifecycle. We believe that this work is a natural starting point to rethink of industry policies, not specific to merger policies.

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