# Optimal Taxation of Paid- and Self-Employment 

A. Bhandari, D. Evans, E. McGrattan, Y. Yao

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## Conventional Approaches to Optimal Taxation

- Mirrlees:
- Few restrictions on fiscal system
- Many modeling shortcuts (eg, heterogeneity, dynamics, GE)
- Wide range of shapes for marginal tax rates
- Ramsey:
- Tight restrictions on fiscal system
- Few modeling shortcuts
- Wide range of magnitudes for marginal tax rates

This paper: Bridge between the two

## Two headaches

- Rich fiscal systems
$\Rightarrow$ high-dimensional choice vector for planner
- Rich GE models
$\Rightarrow$ hard-to-compute but welfare-relevant transitions


## Two contributions

- Methodological
- Fast method to compute transition
- Application
- Find optimal shape of labor wedge in Aiyagari model
- Find implementation with income and consumption taxes


## Findings

- Shape of the labor wedge
- approximately constant
- ignoring transition, optimal wedge is smaller and regressive
- Welfare gains
- Result: gains large when tax on consumption large
- Intuition: consumption tax effectively taxes capital distributions


## Model: Households

Continuum of households consume, supply labor, and save:

$$
V_{t}(a, \epsilon)=\max _{c, n, a^{\prime}} u(c, \ell)+\beta \mathbb{E}_{\epsilon^{\prime} \mid \epsilon}\left[V_{t+1}\left(a^{\prime}, \epsilon^{\prime}\right) \mid \epsilon\right]
$$

subject to

$$
\begin{aligned}
& \left(1+\tau_{c}\right) c+(1+\gamma) a^{\prime}=\left(1+\bar{r}_{t}\right) a+w_{t} \in n-T_{t}^{n}(w \in n) \\
& a^{\prime}, \ell=1-n, \geq 0 \quad n \in[0,1]
\end{aligned}
$$

Labor wedge $\equiv 1-\left(\frac{u_{l} / u_{c}}{w_{t} \epsilon}\right)$

## Model: Corporate-sector Firms

Representative firm maximizes a sum of discounted after-dividend tax dividend flows

$$
v_{c, t}(k)=\max _{n, k}\left(1-\tau_{d}\right) d+\frac{1+\gamma}{1+\bar{r}_{t}} v_{c, t+1}\left(k^{\prime}\right)
$$

subject to

$$
\begin{aligned}
& (1+\gamma) k^{\prime}=\left(1-\delta_{c}\right) k+x \\
& y=A F(k, n) \\
& d=y-w_{t} n-x-\tau_{p}\left(y-w_{t} n-\delta k\right)
\end{aligned}
$$

## Model: Government

Government

- Spends $g$
- Borrows b
- Pays interest at rate $\bar{r}$
- Collects consumption taxes at rate $\tau_{c}$
- Collects labor income taxes with schedule $T^{n}(\cdot)$
- Collects profit taxes with rate $\tau_{p}$
- Collects dividend taxes with rate $\tau_{d}$

To satisfy

$$
\begin{aligned}
g+\left(\bar{r}_{t}-\gamma\right) b & =\tau_{c} \int c_{i t} d i+\int T_{t}^{n}\left(w_{t} \epsilon_{i t} n_{i t}\right) d i+\tau_{p, t}\left(y_{t}-w_{t} n_{t}-\delta k_{t}\right) \\
& +\tau_{d}\left(y_{t}-w_{t} n_{t}-\left(\gamma+\delta_{k}\right) k-\tau_{p}\left(y_{t}-w_{t} n_{t}-\delta k_{t}\right)\right)
\end{aligned}
$$

## Planning problem

From hh optimality, labor wedge depends $\left\{T^{n}(\cdot), \tau_{c}\right\}$

- Choices:

$$
\Upsilon \equiv\left\{T^{n}(\cdot), \tau_{c}\right\}
$$

ensuring revenue neutrality

- Welfare criteria:

$$
W\left(\Omega_{0} ; \Upsilon\right) \equiv \int V_{0}(a, \epsilon ; \Upsilon) d \Omega_{0}
$$

Note:

- Utilitarian weights over individual welfare include transitions


## Fast Method to Compute Transition

- Want to approximate

$$
W\left(\Omega_{0} ; \Upsilon\right) \equiv \int V_{0}(a, \epsilon ; \Upsilon) d \Omega_{0}
$$

- Idea:
- Take Taylor expansion

$$
W(\Omega)=W(\bar{\Omega})+W_{\Omega}(\bar{\Omega}) \cdot(\Omega-\bar{\Omega})+\frac{1}{2} W_{\Omega \Omega}(\bar{\Omega}) \cdot(\Omega-\bar{\Omega}, \Omega-\bar{\Omega})+\ldots
$$

- Need to compute Frechet derivatives $W_{\Omega}, W_{\Omega \Omega}, \ldots$


## Computing Transitions: Frechet Derivatives

$$
W\left(\Omega_{0} ; \Upsilon\right) \equiv \int V_{0}(a, \epsilon ; \Upsilon) d \Omega_{0}
$$

How to proceed?

1. Set $\bar{\Omega}$ to the new steady state for reform $\Upsilon$
2. Define direction $\Delta_{0} \equiv \Omega_{0}-\bar{\Omega}$

Differentiate once

$$
W_{\Omega}(\bar{\Omega}) \cdot \Delta_{0}=\int V_{0}(a, \epsilon ; \Upsilon) d \Delta_{0}+\int V_{0, \Omega}(a, \epsilon ; \Upsilon) \cdot \Delta_{0} d \bar{\Omega}
$$

where $V_{0, \Omega}$ depends on policy function derivatives

$$
\left\{c_{t, \Omega}(a, \epsilon) \cdot \Delta_{0}, n_{t, \Omega}(a, \epsilon) \cdot \Delta_{0}\right\}_{t=0}^{\infty}
$$

Length of transition $\times$ number of nodes to store policy functions $\times$ number of points to store the distribution!

## Three mappings to represent equilibria

## Cast of characters

- $z=(a, \epsilon)$ individual states, $\Omega$ distribution over $z$
- $\tilde{x}(z, \Omega)$ individual policies
- $\tilde{X}(\Omega)$ aggregate variables


## Equilibrium

Optimality

$$
0=F\left(z, \tilde{x}(z, \Omega), \tilde{x}(\Omega), \mathbb{E}\left[\tilde{x}\left(\mathrm{p} \tilde{x}(z, \Omega)+\epsilon^{\prime}, \tilde{\Omega}(\Omega)\right)\right]\right)
$$

Law of motion

$$
\tilde{\Omega}(\Omega)(y)=\iint \iota(\mathrm{p} \tilde{x}(z, \Omega)+\epsilon \leq y) d \operatorname{Pr}(\epsilon) d \Omega(z)
$$

Market clearing

$$
0=R\left(\int \tilde{x}(z, \Omega) d \Omega(z), \tilde{x}(\Omega)\right)
$$

Equilibirum summarized by three mappings: $\{F, R, \tilde{\Omega}\}$

## Computing Policy Function Derivatives: $\left\{x_{t, \Omega}(z) \cdot \Delta_{0}\right\}$

- Express policy function derivatives in terms of $\left\{X_{t, \Omega} \cdot \Delta_{0}\right\}_{t}$

Differentiate individual optimality conditions for $\left\{x_{t, \Omega} \cdot \Delta_{0}, n_{t, \Omega} \cdot \Delta_{0}\right\}_{t}$

$$
0=F\left(z, \tilde{x}\left(z_{t}, \Omega_{t}\right), \tilde{x}\left(\Omega_{t}\right), \mathbb{E}_{t}\left[\tilde{x}\left(\mathrm{p} \tilde{x}\left(z_{t}, \Omega_{t}\right)+\epsilon_{t+1}, \tilde{\Omega}\left(\Omega_{t}\right)\right)\right]\right)
$$

Differentiate law of motion of distribution for $\left\{\Omega_{t, \Omega} \cdot \Delta_{0}\right\}_{t}$

$$
\tilde{\Omega}\left(\Omega_{t}\right)(y)=\iint \iota\left(\mathrm{p} \tilde{x}\left(z, \Omega_{t}\right)+\epsilon \leq y\right) d \operatorname{Pr}(\epsilon) d \Omega_{t}(z)
$$

- With some work, the previous two steps can be done in closed-form (i.e., no numerical/symbolic/automatic differentiation)
- Solve a much smaller linear system in $\left\{X_{t, \Omega} \cdot \Delta_{0}\right\}_{t}$ with market clearing conditions

$$
0=R\left(\int \tilde{x}\left(z, \Omega_{t}\right) d \Omega_{t}(z), \tilde{x}\left(\Omega_{t}\right)\right)
$$

- Same logic extends to higher order derivatives


## Functional Forms

- Period utility: $u(c, \ell)=\frac{\left(c^{\eta} \ell^{1-\eta}\right)^{1-\mu}}{1-\mu}$
- Production: $F(k, n)=A k^{\theta} n^{1-\theta}$
- Skills: $\ln \epsilon_{t}=$ Persistent + iid
- Labor tax schedule:

$$
T^{n}(y)= \begin{cases}\tau_{1}^{n} y-\psi_{1}^{n} & \text { if } y \in\left[0, y_{1}^{n}\right] \\ \tau_{2}^{n} y-\psi_{2}^{n} & \text { if } y \in\left[y_{1}^{n}, y_{2}^{n}\right] \\ \vdots & \vdots \\ \tau_{N-1}^{n} y-\psi_{N-1}^{n} & \text { if } y \in\left[y_{N-2}^{n}, y_{N-1}^{n}\right] \\ \tau_{N}^{n} y-\psi_{N}^{n} & \text { if } y \in\left[y_{N-1}^{n}, \infty\right),\end{cases}
$$

where $\left\{\psi_{i}^{n}\right\}$ chosen so $T^{n}$ is continuous

## Implied Moments for US Parameterization

| Moments | Value |
| :--- | :---: |
| After-tax return on capital | $4 \%$ |
| Business Capital / GDP | 3.5 |
| Government Debt /GDP | 1 |
| Hours | $28 \%$ |
| Labor income /GDP | $64 \%$ |
| Average Labor wedge | $34 \%$ |
| Government Consumption/ GDP | $13 \%$ |
| Labor income Gini | $62 \%$ |
| Wealth Gini | $72 \%$ |

## Optimal labor wedge

Labor Wedge $=1-\left(\frac{1-\tau^{n}}{1+\tau_{c}}\right)$


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## How tightly is the optimum pinned down?

Let $\omega(\Upsilon)$ be the consumption eq. welfare gains relative to $\Upsilon^{U S}$. For some $\delta \in(0,1)$ define

$$
\mathcal{C}(\delta) \equiv\left\{\Upsilon: \omega(\Upsilon) \geq \delta \omega\left(\Upsilon^{O P T}\right)\right\}
$$

Shape of $\mathcal{C}(\delta)$ informative about welfare sensitivity

## Welfare Sensitivity



Darker shades represent tighter welfare bands by setting $\delta \in\{0.75,0.9,0.95\}$

## Welfare Gains from Consumption Tax



## Sources of Welfare Gains

- Recall household budget constraint

$$
a_{t}\left(\frac{1+\bar{r}}{1+\tau_{c}}\right)=\sum_{j}\left(\frac{1+\gamma}{1+\bar{r}}\right)^{j}\left\{c_{t+j}-\text { Labor wedge } \times\left(w \epsilon_{t} n_{t}\right)+\text { Transfers }\right\}
$$

- Taxing consumption like taxing capital income distributions
- Pros: valuable for planner who cares about redistribution
- Cons: households lose ability to smooth and need to cut consumption after the reform to increase savings
- Optimal labor wedge in baseline Aiyagari is approximately constant
- Consumption taxes a powerful tool for redistribution
- More discussion needed about consumption taxes
- Why not tax wealth directly?
- Will there be shifts towards non-market transactions?
- Will there be shifts towards consumption on the job?

