## Optimal Taxation of Paid- and Self-Employment

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## Conventional Approaches to Optimal Taxation

#### Mirrlees:

- Few restrictions on fiscal system
- Many modeling shortcuts (eg, heterogeneity, dynamics, GE)

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Wide range of shapes for marginal tax rates

#### Ramsey:

- Tight restrictions on fiscal system
- Few modeling shortcuts
- Wide range of magnitudes for marginal tax rates

#### This paper: Bridge between the two

### Two headaches

Rich fiscal systems

 $\Rightarrow$  high-dimensional choice vector for planner

#### Rich GE models

 $\Rightarrow$  hard-to-compute but welfare-relevant transitions

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## Two contributions

#### Methodological

Fast method to compute transition

#### Application

- Find optimal shape of labor wedge in Aiyagari model
- Find implementation with income and consumption taxes

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# Findings

Shape of the labor wedge

- approximately constant
- ignoring transition, optimal wedge is smaller and regressive

#### Welfare gains

- Result: gains large when tax on consumption large
- Intuition: consumption tax effectively taxes capital distributions

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### Model: Households

Continuum of households consume, supply labor, and save:

$$V_t(\mathbf{a},\epsilon) = \max_{\mathbf{c},\mathbf{n},\mathbf{a}'} u(\mathbf{c},\ell) + \beta \mathbb{E}_{\epsilon'|\epsilon} \left[ V_{t+1}(\mathbf{a}',\epsilon')|\epsilon \right]$$

subject to

$$(1 + \tau_c)c + (1 + \gamma)a' = (1 + \bar{r}_t)a + w_t \epsilon n - T_t^n(w \epsilon n)$$
  
a',  $\ell = 1 - n, \ge 0$   $n \in [0, 1]$ 

Labor wedge  $\equiv 1 - \left(rac{u_l/u_c}{w_t\epsilon}
ight)$ 

### Model: Corporate-sector Firms

Representative firm maximizes a sum of discounted after-dividend tax dividend flows

$$v_{c,t}(k) = \max_{n,k} (1 - \tau_d) d + \frac{1 + \gamma}{1 + \overline{r}} v_{c,t+1}(k')$$

subject to

$$(1+\gamma)k' = (1-\delta_c)k + x$$
  

$$y = AF(k, n)$$
  

$$d = y - w_t n - x - \tau_p (y - w_t n - \delta k)$$

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### Model: Government

Government

- Spends g
- Borrows b
- Pays interest at rate r̄
- Collects consumption taxes at rate τ<sub>c</sub>
- Collects labor income taxes with schedule  $T^n(\cdot)$
- Collects profit taxes with rate τ<sub>p</sub>
- Collects dividend taxes with rate  $\tau_d$

To satisfy

$$g + (\bar{r}_t - \gamma)b = \tau_c \int c_{it} di + \int T_t^n (w_t \epsilon_{it} n_{it}) di + \tau_{p,t} (y_t - w_t n_t - \delta k_t) + \tau_d (y_t - w_t n_t - (\gamma + \delta_k)k - \tau_p (y_t - w_t n_t - \delta k_t))$$

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## Planning problem

From hh optimality, labor wedge depends  $\{T^n(\cdot), \tau_c\}$ 

Choices:

$$\Upsilon \equiv \{T^n(\cdot), \tau_c\}$$

ensuring revenue neutrality

Welfare criteria:

$$W(\Omega_0;\Upsilon)\equiv\int V_0(a,\epsilon;\Upsilon)d\Omega_0$$

Note:

Utilitarian weights over individual welfare include transitions

## Fast Method to Compute Transition

Want to approximate

$$W(\Omega_0;\Upsilon)\equiv\int V_0(a,\epsilon;\Upsilon)d\Omega_0$$

Idea:

Take Taylor expansion

$$W(\Omega) = W(\overline{\Omega}) + W_{\Omega}(\overline{\Omega}) \cdot (\Omega - \overline{\Omega}) + \frac{1}{2} W_{\Omega\Omega}(\overline{\Omega}) \cdot \left(\Omega - \overline{\Omega}, \Omega - \overline{\Omega}\right) + \dots$$

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Need to compute Frechet derivatives  $W_{\Omega}$ ,  $W_{\Omega\Omega}$ , ...

### Computing Transitions: Frechet Derivatives

$$W(\Omega_0;\Upsilon)\equiv\int V_0(a,\epsilon;\Upsilon)d\Omega_0$$

How to proceed?

- 1. Set  $\overline{\Omega}$  to the new steady state for reform  $\Upsilon$
- 2. Define direction  $\Delta_0\equiv\Omega_0-\overline{\Omega}$

Differentiate once

$$W_{\Omega}(\overline{\Omega}) \cdot \Delta_0 = \int V_0(a,\epsilon;\Upsilon) d\Delta_0 + \int V_{0,\Omega}(a,\epsilon;\Upsilon) \cdot \Delta_0 d\overline{\Omega}$$

where  $V_{0,\Omega}$  depends on policy function derivatives

$$\{c_{t,\Omega}(a,\epsilon)\cdot\Delta_0, n_{t,\Omega}(a,\epsilon)\cdot\Delta_0\}_{t=0}^{\infty}$$

Length of transition x number of nodes to store policy functions x number of points to store the distribution!

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### Three mappings to represent equilibria

#### Cast of characters

- $rac{z}{z} = (a, \epsilon)$  individual states, Ω distribution over z
- $\tilde{x}(z, \Omega)$  individual policies
- $\tilde{X}(\Omega)$  aggregate variables

#### Equilibrium

Optimality

$$0 = F\left(z, \tilde{x}(z, \Omega), \tilde{X}(\Omega), \mathbb{E}\left[\tilde{x}\left(p\tilde{x}(z, \Omega) + \epsilon', \tilde{\Omega}(\Omega)\right)\right]\right)$$

Law of motion

$$ilde{\Omega}\left(\Omega
ight)\left(y
ight)=\int\int\iota\left(\mathsf{p} ilde{x}(z,\Omega)+\epsilon\leq y
ight)\mathsf{d}\mathsf{Pr}(\epsilon)\mathsf{d}\Omega(z)$$

Market clearing

$$0 = R\left(\int \tilde{x}(z,\Omega)d\Omega(z),\tilde{X}(\Omega)\right)$$

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Equilibirum summarized by three mappings:  $\{F, R, \tilde{\Omega}\}$ 

## Computing Policy Function Derivatives: $\{x_{t,\Omega}(z) \cdot \Delta_0\}$

Express policy function derivatives in terms of {X<sub>t,Ω</sub> · Δ<sub>0</sub>}<sub>t</sub>

Differentiate individual optimality conditions for  $\{x_{t,\Omega} \cdot \Delta_0, n_{t,\Omega} \cdot \Delta_0\}_t$ 

$$0 = F\left(z, \tilde{x}(z_t, \Omega_t), \tilde{X}(\Omega_t), \mathbb{E}_t\left[\tilde{x}\left(p\tilde{x}(z_t, \Omega_t) + \epsilon_{t+1}, \tilde{\Omega}(\Omega_t)\right)\right]\right)$$

Differentiate law of motion of distribution for  $\{\Omega_{t,\Omega} \cdot \Delta_0\}_t$ 

$$ilde{\Omega}\left(\Omega_t
ight)(y) = \int \int \iota\left(\mathsf{p} ilde{x}(z,\Omega_t) + \epsilon \leq y
ight) d\mathsf{Pr}(\epsilon) d\Omega_t(z)$$

- With some work, the previous two steps can be done in closed-form (i.e., no numerical/symbolic/automatic differentiation)
- Solve a much smaller linear system in  $\{X_{t,\Omega} \cdot \Delta_0\}_t$  with market clearing conditions

$$0 = R\left(\int \tilde{x}(z,\Omega_t)d\Omega_t(z),\tilde{X}(\Omega_t)\right)$$

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Same logic extends to higher order derivatives

### **Functional Forms**

• Period utility: 
$$u(c, \ell) = \frac{(c^{\eta} \ell^{1-\eta})^{1-\mu}}{1-\mu}$$

• Production: 
$$F(k, n) = Ak^{\theta} n^{1-\theta}$$

- Skills:  $\ln \epsilon_t = \text{Persistent} + \text{iid}$
- Labor tax schedule:

$$\mathcal{T}^{n}(y) = \begin{cases} \tau_{1}^{n}y - \psi_{1}^{n} & \text{if } y \in [0, y_{1}^{n}] \\ \tau_{2}^{n}y - \psi_{2}^{n} & \text{if } y \in [y_{1}^{n}, y_{2}^{n}] \\ \vdots & \vdots \\ \tau_{N-1}^{n}y - \psi_{N-1}^{n} & \text{if } y \in [y_{N-2}^{n}, y_{N-1}^{n}] \\ \tau_{N}^{n}y - \psi_{N}^{n} & \text{if } y \in [y_{N-1}^{n}, \infty), \end{cases}$$

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where  $\{\psi_i^n\}$  chosen so  $T^n$  is continuous

## Implied Moments for US Parameterization

Moments	Value
After-tax return on capital	4%
Business Capital / GDP	3.5
Government Debt /GDP	1
Hours	28%
Labor income /GDP	64%
Average Labor wedge	34%
Government Consumption/ GDP	13%
Labor income Gini	62%
Wealth Gini	72%

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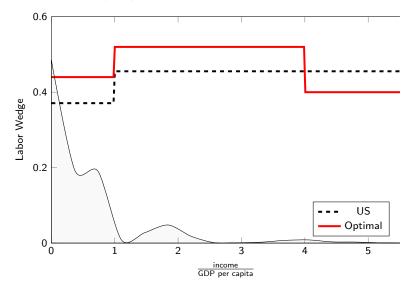
Optimal labor wedge

Labor Wedge = 
$$1 - \left(\frac{1-\tau^n}{1+\tau_c}\right)$$

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Optimal labor wedge

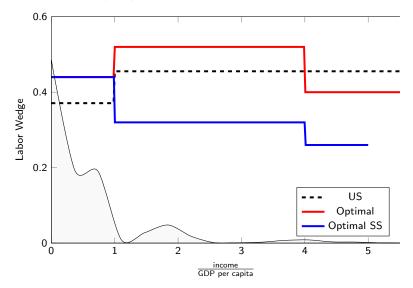
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$$1 - \left(\frac{1-\tau^n}{1+\tau_c}\right)$$



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Optimal labor wedge

Labor Wedge = 
$$1 - \left(\frac{1-\tau^n}{1+\tau_c}\right)$$



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## How tightly is the optimum pinned down?

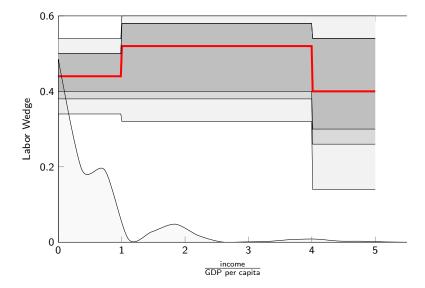
Let  $\omega(\Upsilon)$  be the consumption eq. welfare gains relative to  $\Upsilon^{US}.$  For some  $\delta \in (0,1)$  define

$$\mathcal{C}(\delta) \equiv \left\{ \Upsilon : \omega(\Upsilon) \ge \delta \omega(\Upsilon^{OPT}) \right\}$$

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Shape of  $C(\delta)$  informative about welfare sensitivity

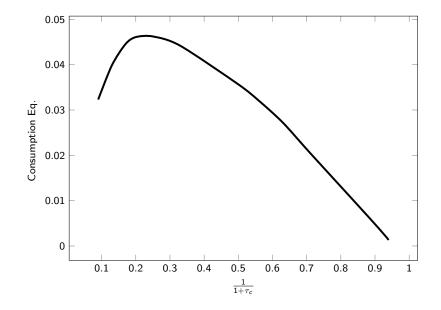
# Welfare Sensitivity



Darker shades represent tighter welfare bands by setting  $\delta \in \{0.75, 0.9, 0.95\}$ 

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## Welfare Gains from Consumption Tax



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## Sources of Welfare Gains

Recall household budget constraint

$$a_t \left(\frac{1+\bar{r}}{1+\tau_c}\right) = \sum_j \left(\frac{1+\gamma}{1+\bar{r}}\right)^j \left\{c_{t+j} - \text{Labor wedge} \times (w\epsilon_t n_t) + \text{Transfers}\right\}$$

Taxing consumption like taxing capital income distributions

- Pros: valuable for planner who cares about redistribution
- Cons: households lose ability to smooth and need to cut consumption after the reform to increase savings

#### Lessons

Optimal labor wedge in baseline Aiyagari is approximately constant

- Consumption taxes a powerful tool for redistribution
- More discussion needed about consumption taxes
  - Why not tax wealth directly?
  - Will there be shifts towards non-market transactions?
  - Will there be shifts towards consumption on the job?